

## Holding of International Reserves with Structural Break: The Case of G7 Countries

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### Abstract

Structural breaks in the data set result changes in the estimated parameters of an econometric model at the break point. So, at the point when data experiences structural break explanatory variables can not accurately predict the future values of the dependent variable. This paper based on the theoretical structure of Wardrobe Theory of International Reserves by Fritz Machlup uses the CUSUM and CUSUM of squares test to scrutinize the validity of a previous study on the propensity of holding reserves for G7 countries. CUSUM and CUSUM of squares recursively trace a break point in the data set and thus help in the identification of the break point. The findings of the study provide guideline on the utility of the study on structural break in international finance.

**Keywords and Phrases:** *Co integration, Chow test, CUSUM and CUSUM Squared test, ADF and ADF\* test*

### 1. Introduction

The earlier theoretical framework of studies on demand for international reserves dealt on the adequacy of reserves used simple criteria such as ratio measure- ratio of reserves to imports. Machlup's (1966) Theory of international reserves established that reserve holding could not be explained through ratio measure. This study based on macro data for 14 countries showed the highest and the lowest ratio in any year between 1949 and 1965 and also the average ratio during the five year period 1961-65. The ratio, as it transpired by empirical estimation failed to convey adequately the need for reserves. Machlup (1966) emphasized that there were none that lend support both on the theoretical and empirical grounds. He emphasized that "if there is (any significance to be attached to this ratio), I would not know of any theory that would show and explain it."

Machlup's (1966) study did not incorporate any regression analysis. However, the idea subsequently generated interest on the study on demand for international reserves through regression analysis. Studies (Clark and Kelly (1970), Frenkel (1980), Saidi (1981), Frenkel and Jovanovic (1981), Edwards (1985), Ben-Bassat and Gottlieb (1992)) using regression technique during 70's and 80's have established the proposition that indeed reserve demand is explained by a well defined demand function for reserves. However, most of the studies based on OLS method suffered from the evils of spurious regression as well as violation of some of the assumptions on the error term. A survey article by Bahmani (2002) offered mixed results on the role of determinants in reserves accumulation.

During eighties, the idea of co integration emanates from the concept of spurious regression as illustrated by Granger and Newbold (1974) opened a new horizon in this estimation spree of reserve demand equation. Engle and Granger (1987) outlined a two step method to deal with individual non-stationary variables in order to study the long run relationship without losing information on the level variables since first differences in variables that render a series stationary may result in loss of information. The method is based on an OLS estimate of the co integrating vector and a unit root test of its residuals. Dickey and

Fuller (1979) provided a statistical device to check for the stationary or non-stationary of data sets through ADF to determine whether a time series is stationary or non-stationary and determine the order of integration to free regression analyses from the evils of spurious regression.

Rahman (2009) in a study on the G7 countries for the period 1973.I -1992.IV on determinants of reserves by using co integration and error correction modeling observed that except for the United Kingdom, the calculated value of the ADF test is larger than the critical value indicating that the residuals are not integrated of the order zero. Therefore, for the United States, Japan, Italy, France, Canada and Germany the null hypothesis of no co integration can not be rejected. This manifests that there exists no long run relationship between reserves as dependent variable and other explanatory variables such as variability measure of the balance of payments and gross domestic product. Thus, propensity of reserve holdings can not be explained by any of the explanatory variables under consideration.

The premise of this study is to check the validity of the results obtained through the co integration and error-correction modeling by introducing the idea of structural break. Data that experience a break when regressed against the probable regressors must show a change in the regression coefficient and thus may not exhibit a long run relationship between the dependent variable and the explanatory variables for the whole length of time.

This study is organized through four sections. Section 2 describes the methodology and Section 3 presents the empirical findings. The study ends with concluding remarks in section 4.

## **2. Methodology**

There are many arguments in the study of demand for holdings of international reserves. The two most common explanatory variables are the real income and the variability measure of the balance of payments. Both variables influence the demand for international reserves positively. International reserves expressed in millions of USD equals the sum of gold, special drawings rights, foreign exchange and the reserve position at the IMF. Gross domestic income expressed in billions of domestic currency is converted into USD using bilateral exchange rate and the U.S. GDP deflator (1987 = 100) is used to convert the nominal variables into real variables. The variability measure of the balance of payments is computed for each quarter by the standard deviation of exports over the preceding 12 quarters and the current quarter.

Findings of the previous study assert that holdings of international reserves cannot be explained by the arguments for USA, Japan, Italy, France, Canada and Germany. U.K is the only exception. Structural break results in parameter invariance. So, a study on the consequences, if any, due to structural break in the data sets may enrich the findings of the previous study. There are many studies to substantiate this point. Perron's (1989) findings may be a case in point where he tested the reliability of the unit root hypothesis for 11 out of 14 macroeconomic data series of Nelson and Plosser (1982). This work was facilitated by Hansen [1992] who pioneered a new avenue in considering structural breaks in cointegrating relations with unknown break points. Gregory and Nason (1992) and Gregory and Hansen (1992) also enriched the findings through further works. These advancements unveil a much wider range of economic application and incorporation of values both for ADF and ADF\* for comparison between conventional ADF test with the ADF\* designed to check for regime shifts. The ADF\* gives the values when there is a break point in

the data set. Gregory and Hansen (1992) have shown that the power of the conventional ADF test falls sharply in the presence of a structural break, an implication that the acceptance region of the null hypothesis of no cointegration (there is no long term relationship between reserves and the arguments) become very large.

One can approach the issue of break point identification with a very precise idea on timing. Chow (1960) and Cooley and Prescott (1976) are some of the examples. The Chow test considers partitioning of data sets into two or more data sets with a specific date of structural break to check whether the coefficient vector may be regarded as constant over the subsets. The Chow test uses the AOV technique and the F test.

Another way is to use the exploratory method. It helps to find the break point when we do not have any clear idea about the break point or we do not make any specific hypothesis on the possible timing of the break point. This approach was first formalized by Brown and Durbin (1975). The following model is estimated recursively to test for the constancy of the regression parameter that examines a number of resulting statistics for the identification of a probable break-point

$$y_t = x_t \beta_t + \mu_t, \quad t = 1, 2, \dots, T$$

where  $y_t$ s are observations on the dependent variable,  $x_t$  is a  $K \times 1$  column vector of non stochastic regressors,  $\beta$  is a vector of regression coefficients, and  $\mu_t \sim N(0, \sigma^2)$ . The disturbances are also assumed to be independent. The null hypothesis is

$$H_0 : \beta_1 = \dots = \beta_t = \beta.$$

Recursive estimation employs first  $K$  observations to estimate  $\beta$ . The sample size is then gradually enlarged by adding one observation at a time, and  $\beta$  is re-estimated. At each step, the estimated  $\beta$  value is used to predict the next value of the dependent variable and a sequence of prediction error. When there are structural shifts, there may be either over or under prediction of the dependent variable and possible discontinuities exist in the size of prediction errors. Dufour (1986) enriched the findings through several modifications. His idea of forward and backward residuals is a major contribution in this field. The two tests CUSUM and CUSUM of squares are designed on this theoretical premise.

The CUSUM test is based on the statistic

$$W_t = \frac{1}{s} \sum_{\tau=k+1}^t w_\tau \quad t = k+1, 2, \dots, T$$

where  $s$  is the standard error of the regression fitted to all  $T$  sample points,  $W_t$  is the cumulative sum and it is plotted against  $t$ . When the  $\beta$  vector remains constant from period to period  $E(W_t) = 0$ , but if  $\beta$  changes  $W_t$  will tend to diverge from the zero mean value line. The significance of any departure from the zero line is assessed by reference to a pair of straight lines, the distance between which increases with  $t$ .

The CUSUM test produces a plot of  $W_t$  against  $t$  and also shows the 5% critical line. The movement of  $W_t$  outside the critical lines is suggestive of parameter instability.

The CUSUM of squares test is based on the statistic

$$S_t = \frac{\sum_{\tau=k+1}^t w_{\tau}^2}{\sum_{\tau=k+1}^T w_{\tau}^2}, \quad t = k+1, 2, \dots, T.$$

The mean value line giving the expected value of this test statistic under the hypothesis of parameter consistency is

$$E(s_t) = \frac{t-k}{T-k}$$

which goes from zero at  $t = k$  to unity at  $t = T$ . The significance of the departure of  $S_t$  from its expected line is assessed by reference to a pair of parallel straight lines around the expected line. The CUSUM of squares test gives a plot of  $S_t$  against  $t$  and shows the mean value line and the pair of 5% critical lines. The movement of  $S_t$  outside the critical lines is suggestive of parameter instability.

Following Brown and Durbin (1975) and using exploratory method of analysis, break points through CUSUM and CUSUM of squares test are conducted for each of the country. The movements of the CUSUM plot outside the 5% critical line from the zero mean value line suggest parameter instability. In CUSUM of squares test, the movement of the plot by reference to a pair of parallel straight lines around the expected line is considered. Movements of the plot outside the critical lines suggest parameter instability. The CUSUM test detects gradual or systematic structural change and the CUSUM of squares test detects random change.

### 3. Empirical Findings

For each country and for each variable (converted into logarithm), the level variables do not converge to a deterministic trend, but the first differenced variable converges to a deterministic trend. This is an indication that the first differenced variables are more prone in reaching stationary than the level variables. Given that each of the variables for each country are integrated of the same order, we run the co integration equation for each country and use the residual to conduct the ADF test. Specially, we test the null hypothesis that the residual is non stationary, i.e., the hypothesis of no-co integration.

The reserve demand equation assumes two independent variables, real income and the variability measure of the balance of payments. Therefore, the use of different variables as the left hand side conditioning variable may yield a different vector of co integration parameters. Following Hall (1986) and Miller (1991), we consider all the possible regressions and report the one with the highest adjusted coefficient of determination, because this procedure minimizes the potential bias in the estimate of the co integration parameter.

For all countries except the United Kingdom, the calculated value of the ADF test is larger than the critical value, indicating that the residuals are not integrated of the order zero. Therefore, for the United States, Japan, Italy, France, Canada and Germany, we can not reject the null hypothesis of no cointegration and conclude that there exist no long run relationships among the variables. Thus propensity of reserve holdings can not be explained by any of the explanatory variables under consideration, i.e., the real income and the variability measure of the balance of payments.

**Table 1:** Testing for regime shifts: ADF versus ADF\*

Country	Variable		R <sup>2</sup>	CRDW <sup>b</sup>	ADF[k]	ADF*
	Dependent	Independent				
UK	LGDP	LRES LVAR	0.88	0.68	-5.1140[1]	-4.9756
USA	LRES	LGDP LVAR	0.72	0.18	-2.6298[2]	-3.2523
Japan	LGDP	LRES LVAR	0.83	0.24	-3.1257[2]	-3.9860
Italy	LGDP	LRES LVAR	0.82	0.33	-3.6277[1]	-3.0679
France	LVAR	LGDP LRES	0.68	0.32	-3.4390[1]	-4.8862
Canada	LGDP	LRES LVAR	0.61	0.20	-3.0941[1]	-4.1087
Germany	LGDP	LRES LVAR	0.67	0.29	-2.4121[1]	-3.9197

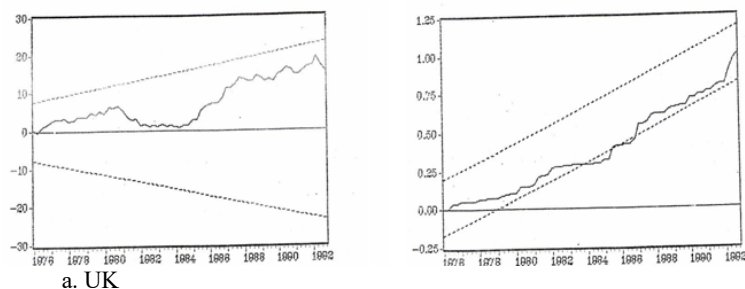
Note: MacKinnon critical value for 80 observations when there are three variables in the co integration equation are -4.2756\*\* at the 5% level and -3.9530\* at the 10% level of significance respectively. . The critical values obtained by Gregory and Hansen (1992) for two regressors at the 5% and at the 10% significant level are -4.92 and -4.69 respectively.

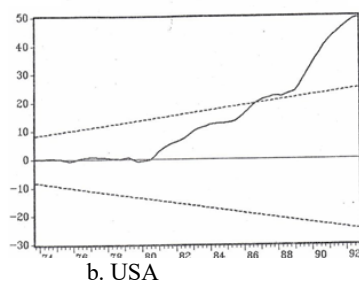
Number of lags[k] in the ADF test is selected by the level of significance of the estimated lag coefficients using the standard t-test.

b. The critical value of D.W. statistic for sample size in the vicinity of 100 observations at the usual 5% level is 0.39. This value is obtained from Engle and Yoo (1987, Table 4).

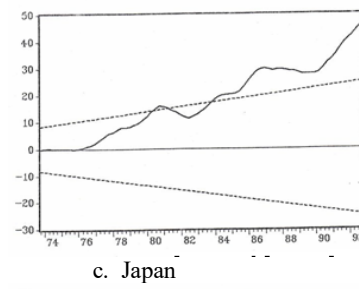
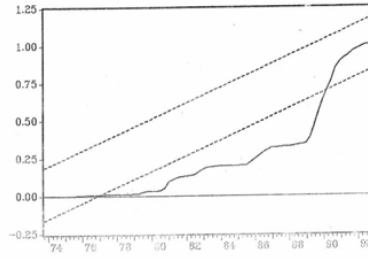
In addition to ADF test, we also use the co integration equation Durbin Watson statistic, CRDW. When the CRDW statistic approaches zero, residuals are nonstationary. As can be seen from Table 1, only in the case of the United Kingdom, the CRDW statistic is larger than the critical value, indicating co integration in this case. The graph both for CUSUM and CUSUM of squares test lies also within the critical line.

To draw a parallelism between the previous findings and the CUSUM test and CUSUM of squares test, plot of CUSUM test and CUSUM of squares test for each country are given in Figure 1 (a-g).

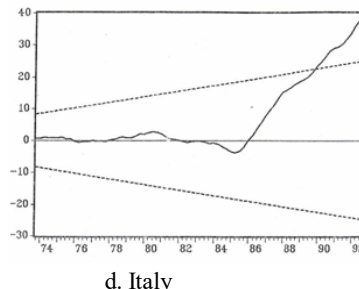
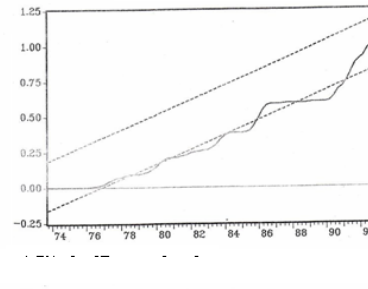




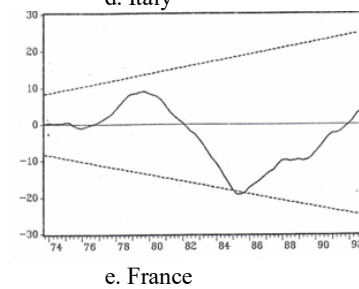
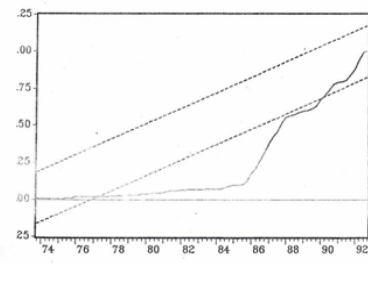
b. USA



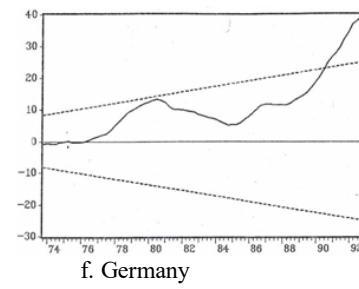
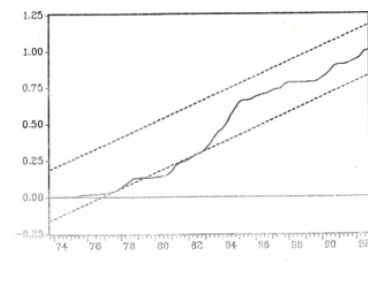
c. Japan



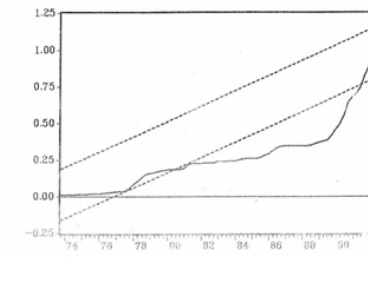
d. Italy

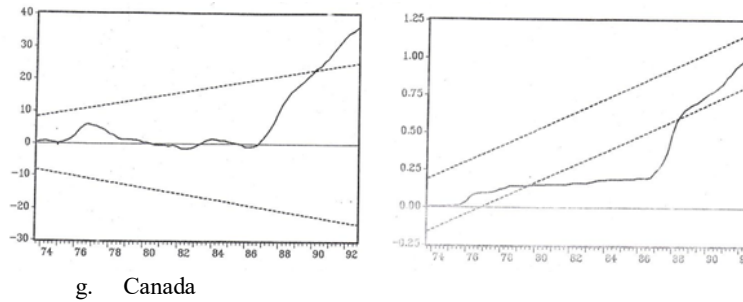


e. France



f. Germany





**Figure 1.** CUSUM and CUSUM of squares at 5% significance level

The plot of the CUSUM and CUSUM of squares tests provide an indication of the probable break in the estimated equation and its consequences on reserve holdings.

For the United Kingdom (Figure 1a), we may reject the null hypothesis using the conventional ADF test and conclude that there exists a long run relationship between reserves and the explanatory variables. The value of the ADF\* test confirms the rejection of the null hypothesis. The plot of CUSUM and CUSUM of squares tests reveal parameter stability and it establishes a long run relationship between reserves and the explanatory variables. For all other countries, though there is no distinct break point, the null cannot be rejected in a straight forward manner as given by the ADF \* values.

For the United States (Figure 1b) until 1980, the series appears to be very smooth, and then there is a gradual shift in one direction and a break around 1986. The variance around the regression is much higher beginning late eighties.

For Japan (Figure 1c) until 1976, the series appears to be very smooth followed by a pattern of gradual upward and downward trends, 1980 and 1984 are two break points. CUSUM of squares resembles the U.K. trend

For Italy (Figure 1d), we observed a very smooth series until 1988 and no visible break point till 1989 though the variance around the regression is much smaller in the first half of the period than in the second.

For France (Figure 1e) the smooth trend exists only for 1974-76, then till 1984 a downward movement with a break in 1985 and for the rest of the period an upward movement. Asymptotic critical values at the 5% and at the 10% significance level are -4.92 and -4.69 respectively. Only in the case of France, we may reject the null of no co integration at the 10% level, not at the conventional 5% level.

Germany (Figure 1f) is a bit stable but experiences a break point around 1990.

For Canada (Figure 1g) the series appears to be very smooth until 1986, but there is a break point which is accentuated gradually after 1990.

In order to have a refined analysis, the data sets for Japan, France and Canada are divided into sub sample with the purpose to carry out the Engle and Granger two-step procedure. The break up of the sample helps checking the residual of the co integration of the equation for each sub sample.

For Japan, the ADF value (-2.0794) of the residual term for sub sample 1973.1-1976.4, and the ADF value (-2.9781) of the residual term for the sub sample indicate acceptance of the null.

For France, the ADF value of the residual term for the period 1985.1-1992.4 [-4.4058] is not significant at the conventional 5% level (the critical value at the 5% level is -4.5347) implying that fixed not reject the null. This reinforces previous findings that the null can not be rejected at the conventional 5% level.

For Canada, the ADF value (-2.6557) of the residual term for the sub sample 1973.1-1984.4 and the ADF value (-3.1078) of the residual term for the sub sample 1985.1-1992.4 indicate acceptance of the null for these two sub sample periods.

#### 4. Conclusions

In summary, we have consistent results for United States, United Kingdom, Italy, and Germany. We accept the null of no-co integration for the United States, Italy, and Germany by both the co integration and structural break methods. For the United Kingdom, we reject the null when tested by both the methods. Introduction of ADF\* test reinforces our findings for all these countries except France, Japan and Canada though the probability is negligible for France. For Canada and Japan the result is inconclusive.

CUSUM test and CUSUM of squares test are often used to test the stability of the demand function for money. But there are not enough studies to test the stability of the demand function for international reserve. This paper, therefore, adds a new dimension in research and its findings provide an important guideline on the implications of structural break in international financial management.

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