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Construction and Analysis of Incomplete Block Change-Over Designs Balanced for First Second and Third-Order Residual Effects

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Abstract

The experiment where experimental unit receives cyclical sequence of the treatments, one at a time, over a certain period of time is known as change-over designs. A series of balanced designs called incomplete block change-over designs has been developed with the help of balanced incomplete block design. The problem, construction, analysis and estimation of direct, first, second-order and third-order residual effects of this design have been presented.

1. Introduction:

In experiments where treatments are applied in sequences to the same experimental units like nutritional experiments related to diary animals, wire houses in storage experiments, at times the effect of certain treatments continue during subsequent periods. This is termed as residual effects. The feature of this design is to estimate the residual effects that enables the estimation of direct effects of the treatment more precisely by eliminating the difference among experimental units from the error variations. These designs have been used in several fields of research, notably for nutrition experiments with dairy cattle, clinical trials in medical research, psychological experiments, long-term agricultural field experiments and bio-assays.

This designs have been developed by several authors. Among them few are Cochran, Autrey and Cannon (1941), Williams (1949, 1950), Patterson and Lucas (1959, 1962), David and Wolock (1965), Rees (1969), Saha (1970), Pigeon and Raghavarao (1987), Cini Varghese (2000) etc.

2. Residual Effects:

Change over designs are widely used designs where treatments are applied in sequences to each experimental unit at times where the effect of certain treatments continue during the subsequent periods and get entangled with the carry over or residual effect. If it persists upto k-th period then it is known as k-th order residual effect. The estimation of residual effect is the distinguishing feature of change-over designs. Residual effects are independent of the treatments applied to the experimental units in the period in which they are observed. Direct effect and residual effects are additive in this case. For example: $t_1, t_2, t_3 : t_2, t_1, t_3$ the measurement of the first order residual effect of treatment t_1 is made in the second period of first sequence and the third period of second sequence. If d is the direct effect of a treatment and r is the first order residual effect r' the second-order residual effect and so on, then the measurements in the two sequences are $d_1, d_2 + r_1, \mathrm{d}_3 + \mathrm{r}_2 + \mathrm{r}_1 : \mathrm{d}_2, \mathrm{d}_1 + \mathrm{r}_2, \mathrm{d}_3 + \mathrm{r}_1 + \mathrm{r}_2$.

3. Condition for Balanced Design:

In order that direct and residual effects of treatments can be estimated without much involment, the designs should be balanced. Designs which possesses the following properties will be balanced:

1. Differences of direct effects of any two treatments are estimated with equal precision.
2. Differences of residual of first order, second-order and third-order effects of any two treatments are estimated with equal precision.

4. Method of Construction:

A BIBD with parameters b =v, r = k = v -1, λ = v -2, where v is any odd number considered. If we associate William's special Latin squares with each block of the above design, then an incomplete block change-over design is obtained with v = b, k = v -1, p = v -1 which will satisfy all condition of balanced designs. Further instead of taking (v – 1) periods if we consider only first p periods, where $P \geq 3$, then also we obtain an IBBCO (Incomplete Block Balanced Change Over) design with parameters v = b, k = v – 1, p = v – 1. This will also satisfy the conditions for balanced design of first-order, second-order and third-order residual effects.

Example:

Let v = 5 be the number of treatments. We consider a BIBD with parameters, b = v = 5, r = k = 4, λ = 3 as follows:

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 0 \\ 3 & 4 & 0 & 1 \\ 4 & 0 & 1 & 2 \end{array} \tag{1}$$

Associating (i) with Williams special Latin squares we obtain a IBBCO design with parameters v = b = 5, k = 4, p = 4, where p is the number of periods.

Sequences

Period	Block I			
1	0	1	2	3
2	1	3	0	2
3	2	0	3	1
4	3	2	1	0

Sequences

Period	Block I			
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Sequences

Period	Block I			
1	2	3	4	0
2	3	0	2	4
3	4	2	0	3
4	0	4	3	2

Sequences

Period	Block I			
1	3	4	0	1
2	4	1	3	0
3	0	3	1	4
4	1	0	4	3

Sequences

Period	Block I			
1	4	0	1	2
2	0	2	4	1
3	1	4	2	0
4	2	1	0	4

If the last period is omitted from the above design we obtain another IBBCO design with v = b = 5, k = 4, p = 3.

4. Analysis of IBBCO Design:

Let us consider an IBBCO design with parameters v = t, b, k and p. i.e. t treatments in b blocks of k units each and in p periods. We assume the following usual fixed-effect model for analysis:

$$y_{ijklmnn_1} = \mu + \alpha_i + \beta_j + \alpha_{kj} + \tau_l + \rho_m + \rho'_n + \rho''_{n_1} + (\pi\beta)_{ij} + \varepsilon_{ijklmnn_l} \qquad (2)$$

i = 1(1) p	(number of periods)
j = 1(1) b	(number of blocks)
k = 1(1) k	(number of units in a block)
l, m, n, n_1 = 1 (1) t	(number of treatments)

where, $y_{ijklmnn_1}$ is the observations corresponding to i-th period and k-th unit in the j-th block receiving treatment l immediately preceded two periods back by treatment n and immediately preceded 3 periods back by treatment n_1.

μ denotes the general mean α_i, i-th period effect, β_j, j-th block effect, α_{kj}, effect of the k-th experimental unit within j-th block, τ_l direct effect of the treatment l, ρ_m, ρ'_n and ρ''_{n_1}, first-order, second-order and third-order residual effect of the treatment m, n and n_1 respectively, $(\alpha\beta)_{ij}$, the interaction effect between i-th period and j-th block and $\varepsilon_{ijklmnn_l}$ is the random error component assumed to be normally distributed with mean zero and constant variance σ^2.

Let the letters t_i', r_i, r_i'', r_i'' be used for the least squares estimates of τ', ρ', ρ'', ρ''' respectively for i = 1, 2, …, t.

Here the restriction are

$$\sum_i \pi_i = \sum_j \beta_j = \sum_k \alpha_{kj} \text{ (for every j)}= \sum_1 \tau_1 = \sum_m \rho_m = \sum_n \rho'_n$$
$$= \sum_{n_1} \rho''_{n_1} = \sum_i (\pi\beta)_{ij} = \sum_j (\pi\beta)_{ij} = 0$$

Let us further define,

(i) Total of the observations of the plots which have treatment i by T_i;

(ii) Total of the observations of the plots which are immediately preceded by the plots receiving treatment i, by R_i;

(iii) Total of the observations from the plots where treatment i is in the preceding two periods back by R'_i;

(iv) Total of the observations from the plots where treatment i situated in the preceding three periods back by R''_i;

(v) Sum of the block totals taking only those blocks which contain treatment i, by $\sum_j B_j^{(i)}$;

(vi) Sum of the unit (sequence) totals taking only those units which contain treatment i (a) in the last period by $\sum S_l^{(i)}$, (b) in the last and last but one period together by $\sum S_l^{\{i\}}$, (c) in the last, last but one period and last but two periods together by $\sum\limits_l S_l^{[i]}$;

(vii) Sum of the period totals taking those blocks which contain treatment i, (a) in the first period by $\sum\limits_j (PB)_{lj}^{(i)}$, (b) in the first (i = 1, 2, j = 1, 2,…b) and second period by $\sum (PB)_{ij}^{(i)}$ i = 1, 2, j = 1, 2, …b.

(c) in the first, second and third period by $\left[\sum (PB)_{ij} + \sum (PB)_{2j} + \sum (PB)_{3j}\right]^{(i)}$

The reduced normal equations for an IBBCO design for estimating direct, first, second and third order residual effects under the model (2) come out as follows:

$$T_i = rkm + k\sum_{j\neq i} b_j^{(i)} + rkt_i + \lambda\sum_{j\neq i} r_j + (\lambda-1)\sum_{j\neq i} r_j' + (\lambda-2)\sum_{j\neq i} r_j''$$

$$R_i = r(k-1)m - k\rho_1 + (k-1)\sum_{j\neq i} b_j^{(i)} - \sum_{k,j} s_{kj} + \lambda\sum_{j=i} t_j + r(k-1)r_i + (\lambda-1)\sum_{j\neq i} r'j + (\lambda-2)\sum_{j\neq i} r_j'' - \sum_j (pb)_{ij}$$

$$R_i' = r(k-2)m - k(\rho_i + \rho_2) + (k-2)\sum_{j\neq i} b_j^{(i)} - \sum_{k.j} s_{kj} + (\lambda-1)\sum_{j\neq i} t_j + (\lambda-1)\sum_{j\neq i} r_j$$
$$+ r(k-2)r_i' + (\lambda-2)\sum_{j\neq i} r_j'' - \sum_{\substack{i=1,2\\ j=1,2\ldots,b}} (PB)_{ij} \qquad \text{(for first two periods)}$$

$$R_i'' = r(k-3)m - k(\rho_1 + \rho_2 + \rho_3) + (k-3)\sum b_j^{(i)} - \sum_{k,j} s_{kj} + (\lambda-2)\sum_{j\neq i} t_j$$
$$(\lambda-2)\sum_{j\neq i} r_j + (\lambda-2)\sum_{j\neq i} r_j' + r(k-3)r_i'' - \sum_{\substack{i=1,2,3\\ j=1,2,..,b}} (Pb)_{ij}$$

$$\sum_j B_j^{(i)} = rk^2m + k^2\sum_j b_j^{(i)} + rkt_i + \lambda k\sum_{j\neq i} t_j + r(k-1)r_i + \lambda(k-1)\sum_{j\neq i} r_j$$
$$r(k-2)r_i + \lambda(k-2)\sum_{j\neq i} r_j' + r(k-3)r_j'' + \lambda(k-3)\sum_{j\neq l} r_j''$$

$$\sum_l S_l^{(i)} = rkm + k\sum_j b_j^{(i)} + k\sum_{kj} s_{kj} + kt_i + \lambda\sum_{j\neq i} t_j + \lambda\sum_{j\neq i} r_j$$
$$+(\lambda-1)\sum_{j\neq 1} r_j^i + (\lambda-2)\sum_{j\neq 1} r_j^{''}$$

$$\sum_l S_l^{\{i\}} = 2rkm + 2k\sum_j b_j^{\{i\}} + k\sum_{k,j} s_{kj} + 2kt_i + 2\lambda\sum_{j\neq i} t_j + kr_i$$
$$+(2\lambda-1)\sum_{j\neq i} r_j + (2\lambda-2)\sum_{j\neq i} r_j' + (2\lambda-4)\sum_{j\neq i} r_j''$$

$$\sum_{l} S_l^{[i]} = 3rkm + 3k\sum_{j} b_j^{[i]} + k\sum_{k,j} s_{kj} + 3kt_i + 3\lambda\sum_{j\neq i} t_j + 2kr_i +$$

$$(3\lambda-2)\sum_{j\neq i} r_j + kr_i' + (3\lambda-4)\sum_{j\neq i} r_j' + (3\lambda-6)\sum_{j\neq i} r_j''$$

$$\sum_{j}(PB)_{1j}^{(i)} = rkm + k^2\rho_1 + k\sum_{j} b_j^{(i)} + kt_i + \lambda\sum_{j\neq i} t_j + k\sum_{j}(P_b)_{1j}$$

$$\left[\sum_{j}(PB)_{1j} + \sum_{j}(PB)_{2j}\right]^{(i)} = 2rkm + k^2(p_1+p_2) + 2k\sum_{j} b_j^{(i)} + 2kt_i$$

$$+2\lambda\sum_{j\neq i} t_j + kr_i + \lambda\sum_{j\neq i} r_j + k\sum_{\substack{i=1,2\\ j=1,2\ldots,b}} (pb)_{ij}$$

$$\left[\sum_{j}(PB)_{1j} + \sum_{j}(PB)_{2j} + \sum_{j}(PB)_{3j}\right]^{(i)} = 3rkm + k^2(p_1+p_2+p_3) + 3k\sum_{j} b_j + 3kt_i$$

$$+2kr_i + 2\lambda\sum_{j\neq i} r_j + kr_i' + \lambda\sum_{j\neq i} r_j' + k\sum_{\substack{i=1,2,3\\ j=1,2,\ldots,b}} (pb)_{ij} \tag{3}$$

5. Eliminating block effect, sequence effect, period effect, the reduced normal equations become:

$$P_i = C\{r(k-1)+\lambda\}t_i - C\{[r(k-1)+\lambda]/k\}r_i$$
$$-C\{[r(k-3)+2\lambda]/k\}r_i' - C\{[r(k-5)+3\lambda]/k\}r_i''$$

$$Q_i = -C\{[r(k-1)+\lambda]/k\}t_i + C\left[\left(k^2-k-1\right)/k^2\right]\{r(k-1)+\lambda\}r_i$$
$$-C\left\{\left[\left(k^2-k-2\right)(r-\lambda)+k(\lambda-1)(k+1)\right]/k^2\right\}r_i'$$
$$-C\left\{\left[\left(k^2-2k-3\right)(r-\lambda)+k(\lambda-2)(k+1)\right]/k^2\right\}r_i''$$

$$M_i = -C\{r[(k-3)+2\lambda]/k\}t_i - C\left\{\left[\left(k^2-k-2\right)(r-\lambda)+k(\lambda-1)(k+1)\right]/k^2\right\}r_i$$
$$+C\left\{\left[r(k-2)\left(k^2-k-2\right)+\lambda\left(k^2-4\right)-2k(\lambda-2)\right]/k^2\right\}r_i'$$
$$-C\left\{\left[\left(k^2-k-6\right)(r-\lambda)+k(\lambda-2)(k+2)\right]/k^2\right\}r_i''$$

$$\begin{aligned} N_i = &-C\left\{\left[r(k-5)+3\lambda\right]/k\right\}t_i - C\left\{\left[\left(k^2-2k-3\right)(r-\lambda)+k(\lambda-2)(k+1)\right]/k^2\right\}r_i \\ &-C\left\{\left[\left(k^2-k-6\right)(r-\lambda)+k(\lambda-2)(k+2)\right]/k^2\right\}r_i' \\ &+C\left\{\left[r(k-3)\left(k^2-k-3\right)+\lambda\left(k^2-9\right)-3k(\lambda-2)\right]/k^2\right\}r_i'' \end{aligned}$$

where, $P_i = T_i - (1/k)\sum_j B_j^{(i)}$

$$Q_i = R_i - \left[(k+1)/k^2\right]\sum_j B_j^{(i)} + (1/k)\sum_l S_l^{(i)} + \frac{1}{k}\sum_j (PB)_{1j}^{(i)}$$

$$M_i = R_i' - \left[(k+2)/k^2\right]\sum_j B_j^{(i)} + (1/k)\sum_l S_l^{\{i\}} + \frac{1}{k}\left[\sum_j (PB) + \sum_j (PB)_{2j}\right]^{(i)}$$

$$N_i = R_i'' - \left[(k+3)/k^2\right]\sum_j B_j^{(i)} + \left(\frac{1}{k}\right)\sum_l S_l^{[i]} + \left(\frac{1}{k}\right)\left[\sum_j (PB)_{1j} + \sum_j (PB)_{2j} + \sum_j (PB)_{3j}\right]^{(i)}$$

………..(4)

and C = 1, when v is even and C = 2, when v is odd.

To find out direct effect first-order, second-order third-order residual effects these normal equations can be written as in matrix form:

$$\begin{bmatrix} P_i \\ Q_i \\ M_i \\ N_i \end{bmatrix} = \begin{bmatrix} C\{r(k-1)+\lambda\} & -C\{[r(k-1)+k]/k\} & -C\{[r(k-3)+2k]/k\} & -C\{[r(k-5)+3\lambda]/k\} \\ -C\{[r(k-1)+\lambda]/k\} & C\left[\left(k^2-k-1\right)/k^2\{r(k-1)+\lambda\}\right] & \begin{gathered} -C\{[(k^2-k-1)(r-\lambda) \\ +k(\lambda-1)(k+1)]\}/k^2 \end{gathered} & \begin{gathered} -C\{[(k^2-2k-3)(r-\lambda) \\ +k(\lambda-2)(k+1)]\}/k^2 \end{gathered} \\ -C\{[r(k-3)+2\lambda]/k\} & \begin{gathered} -C\{[(k^2-k-2)(r-\lambda) \\ +k(\lambda-1)(k+1)]\}/k^2 \end{gathered} & \begin{gathered} C\{[r(k-2)(k^2-k-2) \\ +\lambda(k^2-4)-2k(\lambda-2)]\}/k^2 \end{gathered} & \begin{gathered} -C\{[(k^2-5-6)(r-\lambda) \\ +k(\lambda-2)(k+2)]\}/k^2 \end{gathered} \\ -C\{[r(k-5)+3\lambda]/k\} & \begin{gathered} -C\{[(k^2-2k-3)(r-\lambda) \\ +k(\lambda-2)(k+1)]\}/k^2 \end{gathered} & \begin{gathered} -C\{[(k^2-k-6)(r-\lambda) \\ +k(\lambda-2)(k+2)]\}/k^2 \end{gathered} & \begin{gathered} -C\{[r(k-3)(k^2k-3) \\ \lambda(k^2-9)-3k(\lambda-2)]\}/k^2 \end{gathered} \end{bmatrix} \begin{bmatrix} t_i \\ r_i \\ r_i' \\ r_i'' \end{bmatrix}$$

………(5)

Now solving the equation (6) and assuming C = 1, the estimates of direct effect, first-order, second-order and third-order residual effects are as follows:

$$\hat{t}_i = \frac{1}{\Delta}\left[AP_i + BQ_i + CM_i + DN_i\right]$$
$$\hat{r}_i = \frac{1}{\Delta}\left[BP_i + EQ_i + FM_i + GN_i\right]$$
$$\hat{r}_i' = \frac{1}{\Delta}\left[CP_i + FQ_i + IM_i + JN_i\right]$$
$$\hat{r}_i'' = \frac{1}{\Delta}\left[DP_i + GQ_i + JM_i + LN_i\right] \qquad \text{.........} (6)$$

where A, B, C, D, E, F, G, I, J, L are the co-factors and Δ is the determinant of the above coefficient matrix.

From equations (6) the estimates of variances are as follows:

$$v\left(t_i - t_j\right) = 2A\sigma^2 / \Delta$$
$$v\left(r_i - r_j\right) = 2E\sigma^2 / \Delta$$
$$v\left(r' - r_j'\right) = 2I\sigma^2 / \Delta$$
$$r\left(r_i'' - r_j''\right) = 2L\sigma^2 / \Delta \qquad i \neq j = 1, 2, ..., v$$

Following Lucas and Patterson (1962), the efficiency factors for direct, first, second and third order residual effects are as follows:

$$Ed = \left[2\sigma^2 / nk\right] / \operatorname{var}\left(t_i - t_j\right) = \frac{\Delta}{Ank}$$
$$Er = \left[2\sigma^2 / nk\right] / \operatorname{var}\left(r_i - r_j\right) = \frac{\Delta}{Enk}$$
$$Esr = \left[2\sigma^2 / nk\right] / \operatorname{var}\left(r' - r_j'\right) = \frac{\Delta}{Ink}$$
$$Etr = \left[2\sigma^2 / nk\right] / \operatorname{var}\left(r_i'' - r_j''\right) = \frac{\Delta}{Lnk}$$

6. Conclusion:

A class of design called incomplete block balanced change-over designs has been obtained by associating balanced incomplete block design to special Latin squares of Williams (1949). The incomplete block change-over design of the new class have got the desirable features such as simplicity of construction and provision for estimation of direct effect, first-order, second-order and third-order residual effects. The previous works relating to more than second-order residual effect are very much limited. But in most of the countries including Bangladesh where multiple cropping systems are available the second-order and third-order residual effect cannot be ignored. So the investigations made in this study will estimate the different effects of the treatments more accurately.

Table 1: Analysis of Variance of IBBCO Design

Variation due to	Degrees of freedom	S. S.	M. S. S.
Blocks	b – 1	+	
Periods	p – 1	+	
Blocks x Periods	(b – 1) (p – 1)	+	
Units within blocks	b (k – 1)	+	
Direct effects	v – 1	$\sum_{i=1}^{v} t_i p_i$	
First-order residual effects	v – 1	$\sum_{i=1}^{v} r_i Q_i$	
Second-order residual effects	v – 1	$\sum_{i=1}^{v} r_i' M_i$	
Third-order residual effects	v -1	$\sum_{i=1}^{v} r_i'' N_i$	$S^{2*}e = \frac{s^2 e}{error\, d.f.}$
Error	NP – b (P+k-1) – 3 (v-1)	$S^2 e$ (by subtraction)	
Total	NP -1		

+ Calculated in the usual way

$s^2 e$ is an estimate of σ^2

N is the total number of units (sequences) in the design.

References

W. G. Cochran, K. M. Autrey and C. Y. Cannon, C. Y. (1941): A double change-over design for dairy cattle feeding experiments, Jour Dairy, Sci., 24, 937-951.

H. A. David and F. M. Wolock (1965): Cyclic designs, Ann. Math. Statistics, 36, 1526.

Patterson, H. D. and Lucas, H. L. (1959): Extra-period change-over designs. Biometrics, 15(i), 116.

H. D. Patterson and H. L. Lucas (1962): Change-over designs, North Carolina Agri. Expt. Sta. Tech. Bull., No. 147.

J. G. Pigeon and D. Raghavarao (1987): Crossover designs for comparing treatments with a control, Biometrika, 74(2), 321-328.

D. H. Rees (1969): Some observations on change-over trials, Biometrics, 25(2), 413.

G. M. Saha (1970): Some contributions to design and analysis of experiments involving sequences of treatments, Unpublished Ph. D. thesis, Institute of Agricultural Research Statistics, New Delhi.

C. Varghese, B. Vijaya, and V. K. Shrma (2000): Cross-over designs for comparison of two treatments in presence of residual effects, Ind. J. Anim. Sci., 70(8), 862-865.

E. J. Williams (1949): Experimental designs balanced for the estimation of residual effects of treatments. Austral, Jour. Sci., Res. Ser. A, 2, 149.

E. J. Williams (1950): Experimental designs balanced for pairs of residual effects, Austral, Jour. Sci. Res. A, 3, 351.